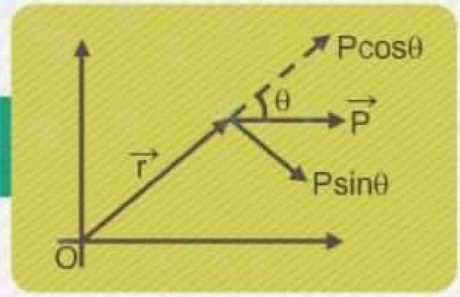
ANGULAR MOMENTUM

PART I

1 ANGULAR MOMENTUM OF A PARTICLE ABOUT A POINT

$\vec{L} = \vec{r} \times \vec{P} \Rightarrow L = rP\sin\theta$



2 ANGULAR MOMENTUM OF A RIGID BODY ROTATING ABOUT A FIXED AXIS

$L = I\omega$

Here, I is the moment of inertia of the rigid body about axis.

3 CONSERVATION OF ANGULAR MOMENTUM

The law of conservation of angular momentum states that when no external torque acts on an object, no change of angular momentum will occur.

Since $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$. Now if, $\vec{\tau}_{net} = 0$, then $\frac{d\vec{L}}{dt} = 0$, so that $\vec{L}$ = constant.

4 ANGULAR IMPULSE

The angular impulse of a torque in a given time interval is defined as $\int_{t_1}^{t_2} \vec{\tau}\, dt$

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

UNIFORM PURE ROLLING

Pure rolling means no relative motion (or no slipping at point of contact between two bodies.)

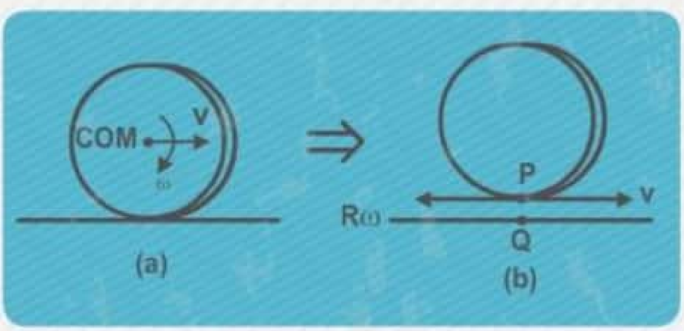
$V_P = V_Q$ or $V - R\omega = 0$ or $V = R\omega$

If $V_P > V_Q$ or $V > R\omega$, the motion is said to be forward slipping and if

$V_P < V_Q < R\omega$, the motion is said to be backward slipping.

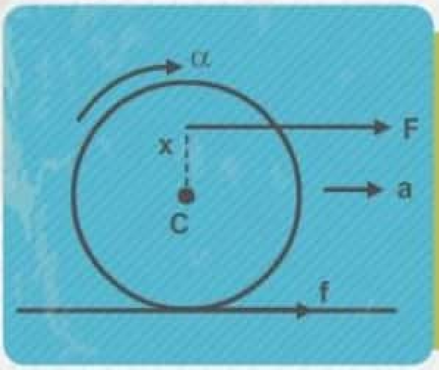
The condition of pure rolling on a stationary ground is,

$a = R\alpha$



1 PURE ROLLING WHEN FORCE F ACT ON A BODY

Suppose a force F is applied at a distance x above the centre of a rigid body of radius R, mass M and moment of inertia CMR^2 about an axis passing through the centre of mass. Applied force F can produces by itself a linear acceleration a and an angular acceleration α.



a = liner acceleration , α = angular acceleration from linear motion

$$F + f = Ma$$

From rotational motion : $Fx - fR = I a$

$$a = \frac{F(R+X)}{MR(C+1)}, \quad f = \frac{F(x-RC)}{R(C+1)}$$

2 PURE ROLLING ON A INCLINED PLANS

A rigid body of radius R, and mass m is released at rest from height h on the incline whose inclination with horizontal is θ and assume that friciton is sufficient for pure rolling then,

$a = \alpha R$ and $v = R\omega$

ω = Angular Velocity

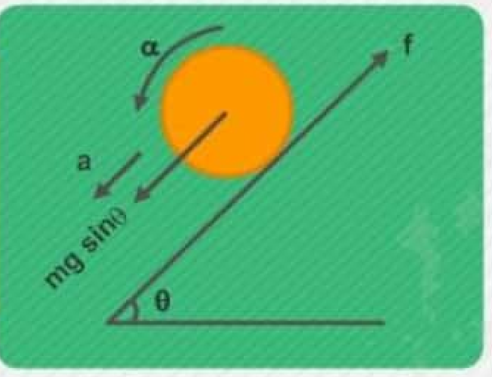
α = Angular Acceleration

Linear Acceleration.

$$a = \frac{g\sin\theta}{1+C}$$

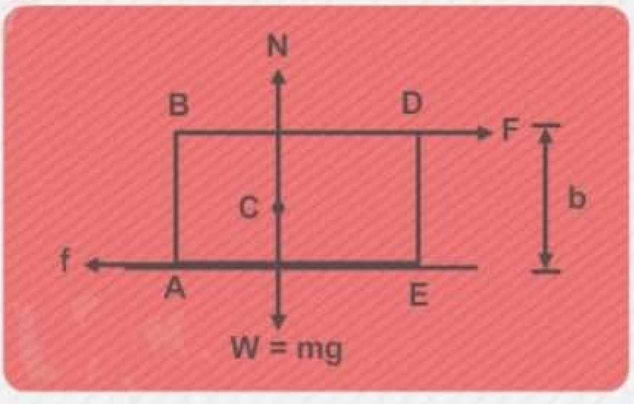
C = Center of Mass

So, body which have low value of C have greater acceleration.



TOPPLING

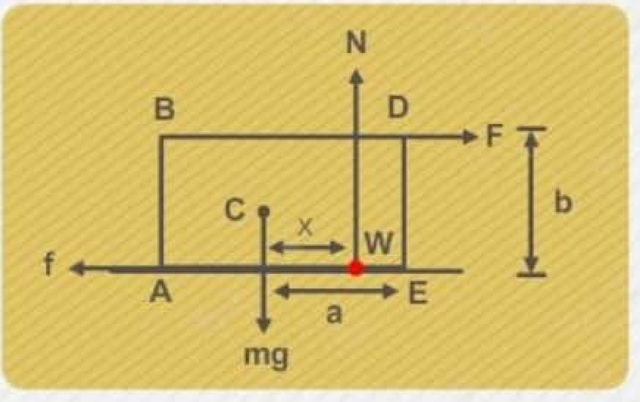
Torque about E



Balancing Torque at E

$$Fb = (mg)a \implies a = \frac{Fb}{mg}$$

Torque about W



Balancing Torque at W

$Fb + N(a - x) = mg\,a$

if $x = a$

$$F_{max}\,b = mga \implies F_{max} = \frac{mga}{b}$$